Exercise 25

In Example 6 we considered a bacteria population that doubles every hour. Suppose that another population of bacteria triples every hour and starts with 400 bacteria. Find an expression for the number n of bacteria after t hours and use it to estimate the rate of growth of the bacteria population after 2.5 hours.

Solution

Let the population be $n(t)$. Since it starts at 400 and triples every hour,

$$n(0) = 400$$

$$n(1) = 3n(0) = 3(400)$$

$$n(2) = 3n(1) = 3[3(400)] = 3^2(400)$$

$$n(3) = 3n(2) = 3\{3[3(400)]\} = 3^3(400)$$

$$\vdots$$

$$n(t) = 3^t(400).$$

Take the derivative to obtain the rate of growth per hour.

$$\begin{aligned}\frac{dn}{dt} &= \frac{d}{dt}[3^t(400)] \\ &= 400\frac{d}{dt}(3^t) \\ &= 400\frac{d}{dt}(e^{\ln 3^t}) \\ &= 400\frac{d}{dt}(e^{t\ln 3}) \\ &= 400(e^{t\ln 3})\cdot\frac{d}{dt}(t\ln 3) \\ &= 400(e^{\ln 3^t})\cdot(\ln 3) \\ &= 400(\ln 3)3^t\end{aligned}$$

Therefore, after 2.5 hours, the rate of growth is

$$\frac{dn}{dt}(2.5) = 400(\ln 3)3^{2.5} \approx 6850\ \frac{\text{bacteria}}{\text{hour}}.$$